Table 1 shows that the [3, 3] approximation ensures a solution which is accurate to 1 part in 10^{-4}. It should be noted that this approximation also gives the same accuracy for a function (3) containing only $V_2(r)$ [3]. Hence, including $V_1(r)$ in the function (3) does not change the order or the approximation in the numerical solution of the Lippmann–Schwinger equation for the range of p, p_1, and k^2 values indicated.

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DETERMINATION OF THE ELECTRODYNAMIC AND THERMAL FLUCTUATION CHARACTERISTICS OF A BICONICAL CAVITY

A. M. Andrusenko, V. F. Kravchenko, and V. A. Solodukho

UDC 621.372.413.017.71:621.391.822.2

A method is proposed for the design of a biconical cavity with finite wall conductivity. The quality of the resonance volume, the temperature field in its walls, and also the level of natural fluctuational thermal radiation are determined.

A number of papers, which are mainly experimental in nature [1-3], are devoted to irregular limit cavities. Theoretical computations of high-quality oscillation systems of similar nature have also been performed [4]. However, the demands of the practice of accurate measurements by using volume resonance apparatus require a strict method of determining the electrodynamic and noise properties of a biconical cavity, its average temperature over the volume, the thermal coefficient, and the maximum allowable dissipation power. The computation of these characteristics is the aim of this paper.

This problem reduces to the solution of the Maxwell equation (rot = curl)

$$\operatorname{rot}\mathbf{E}(\mathbf{r}) = -ik\mathbf{H}(\mathbf{r}), \quad \operatorname{rot}\mathbf{H}(\mathbf{r}) = ik\varepsilon\mathbf{E}(\mathbf{r}) \tag{1}$$

and heat-conduction equation

$$\Delta T(\mathbf{r}) - \frac{1}{\varkappa} W(\mathbf{r}) = 0. \tag{2}$$

The dissipative function $W(\mathbf{r})$ ($\varkappa$ is the coefficient of thermal conductivity of the cavity wall material) has the form [5]

$$W(\mathbf{r}) = -\frac{c}{8\pi}\operatorname{Re}\operatorname{div}[\mathbf{E}(\mathbf{r})\times\mathbf{H}(\mathbf{r})]. \tag{3}$$

We determine the thermal and electrodynamic characteristics of the cavity under investigation in the approximation of given thermal sources and temperature, respectively. Such a linearization of the system of equations (1)-(2), which corresponds physically to neglecting the mutual influence of the temperature and the complex conductivity of the cavity walls, is admissible in the following cases: relatively low power level of the working microwave field and weak dependence of the electrodynamic parameters of the cavity wall material on the temperature.

Khar'kov State Scientific-Research Institute of Metrology. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 2, pp. 332-338, August, 1977. Original article submitted July 7, 1976.

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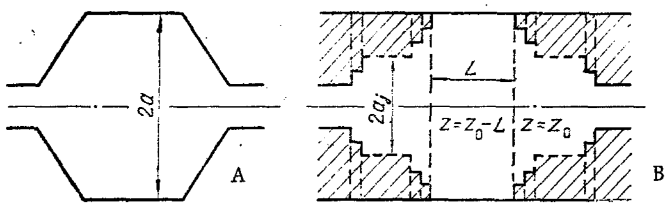

Fig. 1. Biconical cavity (A) and its approximating volume resonance system (B).

The approach used in all the remaining cases results in somewhat exaggerated values of the ultimately allowable microwave power level of the field to be determined in the cavity.

1. In solving the electrodynamic problem, we approximate the geometry of the initial cavity (Fig. 1A) to that when the generators in the section φ = const are a step structure (Fig. 1B). An element in the latter is a coaxial dielectric washer of finite width, whose external radius is a, while the internal radius is a_j ($j = 1, 2, 3, \ldots, n$). The washers are placed in an ideally conducting waveguide. The number of washers n in each of the cavity sides is determined from the condition of the maximum allowable error in the result.

Let us investigate the case of a field distributed symmetrically in φ. Let S_{mn} ($m, n = 1, 2, 3, \ldots$) be the n-th column of the reflection matrix of waveguide waves for each of the nonregular cavity surfaces. We find expressions for the elements of this matrix by using the method of rereflection [7] according to the known solution of the reference problem [6]. Assuming S_{mn} known, we obtain a dispersion equation to determine the eigenfrequencies of such a step cavity.

We consider the domain of the edge boundary an inhomogeneity with known characteristics in the plane $z = z_0$ and $z = z_0 - L$. Then the oscillation system under investigation has the form of a rectangle with perfectly conducting surfaces $r = a$ and known diffraction characteristics at $z = z_0 - L$, $z = z_0$ in the section. The field of such a cavity is represented by a spectrum of direct and reverse H_{0p} waves [8]:

$$E_\varphi = \sum_{m=1}^{\infty} \{A_m \exp[-ih_m(z-z_0)] + B_m \exp[ih_m(z-z_0+L)]\} J_1\left(\frac{\lambda_m}{a} r\right), \tag{4}$$

where $h_m = \sqrt{k^2 - (\lambda_m/a)^2}$; λ_m are roots of the first-order Bessel function, and k is the free space wave number.

Let us consider processes in the $z = z_0$ plane. A set of waveguide waves whose amplitude is denoted by B_m comes in on its left. Waves with amplitude A_m depart from this plane in the $z < 0$ direction. Hence, the following operator equality can be written:

$$(A_m) = ((S_{mn}))(B_n g_n), \tag{5}$$

where $g_n = \exp(ih_n L)$.

By virtue of symmetry of the structure for the $z = z_0 - L$ plane

$$(B_m) = ((S_{mn}))(A_n g_n). \tag{6}$$

That $A_m = \pm B_m$ follows from a direct comparison of (5) and (6).

The upper sign corresponds to the presence of symmetric types of oscillations relative to the $z = z_0 - L/2$ plane, since the magnetic wall in this plane does not change the picture of the field in the cavity. Asymmetric oscillations correspond to the lower sign, since an electrical wall placed in the plane of symmetry introduces no distortions.

According to (4)-(6), the dispersion equation to determine the eigenfrequencies of the oscillator system takes the form

$$\det |E - Sg| = 0. \tag{7}$$

Here E is the unit matrix.

The solution of (7) in combination with (4) yields a complete representation of the field behavior in the cavity under investigation and permits the determination of its quality $Q = \mathrm{Re}\,\omega / 2\,\mathrm{Im}\,\omega$.

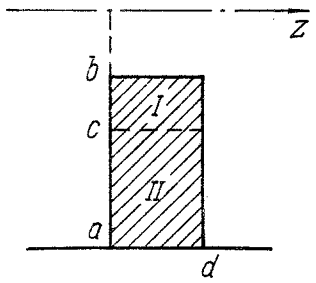

Fig. 2. One of the coaxial dielectric washers comprising the step structure under investigation (the section φ = const).

The passage to the limit $L \to 0$ in (7) results in elimination of that part of the cavity surface which possesses perfect conductivity. In this case the complex dielectric permittivity of the elementary inhomogeneities assures assignment of the finite conductivity of the cavity walls.

2. An electromagnetic field, which penetrates and heats the cavity walls, originates in the system upon excitation of the resonant cavity (Fig. 1B). The temperature sources $W(\mathbf{r})$ in (3) are characterized completely by the intensity of the dielectric losses, which are equivalent to the heat being liberated per unit volume, on the average, during a period of field oscillation:

$$W(\mathbf{r}) = \frac{\omega \operatorname{Im} \varepsilon}{8\pi} \left| \mathbf{E}(\mathbf{r}) \right|^2. \tag{8}$$

The cavity field is described by (4) with known amplitudes of the harmonics A_m and B_m upon excitation of the oscillatory system by H_{0p} waves. This field impregnates the cavity walls and is determined completely within each washer by the E_φ-th component of the electrical field vector (6) (the region $z > 0$) except that still another set of analogous waves being propagated in the direction $z < 0$ is added.

Let us determine the temperature $T(r, z)$ of the cavity side walls under the condition that the distribution of the H_{0p} wave field in the walls is known; i.e., the dissipative function $W(r, z)$ in (8) is defined. The solution of the boundary-value problem with Newton boundary conditions is difficult for the step structure (Fig. 1B). Hence, we shall seek the solution of the heat-conduction equation (2) for each individual coaxial washer (Fig. 2) under the condition of no heat flux on adjoining surfaces of neighboring washers with boundary conditions of the third kind on the free washer surface within the resonant cavity (temperature of the filler gas T_0) and the condition that the temperature T_1 is constant for $r = a$. Such a description of the temperature fields corresponds to the real nature of thermal processes in a step structure. Indeed, for small cavity cone angles the heat exchange between adjacent rings is insignificant, since heating occurs primarily from the inner end-face surface of the washer (region I) and the direction of the temperature gradient is almost radial. The estimates carried out for the magnitude of the longitudinal and radial heat fluxes in irregular domains show that neglecting the longitudinal temperature gradient in the case of the dielectric cavity walls results in an additional error in the result:

$$\frac{T_c - T_1}{T_c + T_1} \left(\frac{\operatorname{tg}\theta}{2\sqrt{2}} \right)^2 100\%,$$

where

$$T_c = \frac{1}{D} \int_0^D T(y)\, dy; \quad y = r \sin\theta + z \cos\theta;$$

θ is the angle between the generator of the irregular cavity surface and the longitudinal axis; and D is the length of the generator. For walls of pure metals, the error is $\rho (T_c/T_1)^{1/3}$ times less than that mentioned for dielectric materials (ρ is the ratio between the thermal-coefficients of the dielectric and the metal).

For slopes of the cavity side walls with its longitudinal axis almost $\pi/2$, the physical model considered does not reflect the actual behavior of the heat fields in the cavity walls. In this case, the planes $r_j = a_j$ rather than the planes z = const must be the section planes with preferred temperature gradient, so that the elementary inhomogeneities are coaxial thin-walled cylinders of different radii.

Solutions of the boundary-value problems are sought analogously in both cases (small and almost $\pi/2$ cone angles). Let us investigate the case of a radial partition into elementary domains (Fig. 2). The division into sections I and II is performed so that the boundary conditions on each boundary of the separate parts would be identical in nature. For $r = c$ the heat fields of domains I and II satisfy the conjugate conditions (9). We seek the solution of the problem posed in the form of series expansions in the eigenvalues of the Sturm–Liouville problem for each of the domains:

$$T(r,z)=\begin{cases} T_0+\sum\limits_{n=1}^{\infty}u_n(r)\left(\cos\mu_n z+\dfrac{h}{\mu_n}\sin\mu_n z\right), & \begin{matrix} b\leqslant r\leqslant c,\\ 0\leqslant z\leqslant d,\end{matrix}\\ T_1+v_0(r)+\sum\limits_{n=1}^{\infty}v_n(r)\cos\nu_n z, & \begin{matrix} c\leqslant r\leqslant a,\\ 0\leqslant z\leqslant d,\end{matrix}\end{cases} \tag{9}$$

where μ_n are solutions of the equation $\tan\mu d = h/\mu$; $\nu n = \pi_n/d$; and h is the coefficient of heat emission by the cavity walls into its cavity.

Substituting T(r, z) from (9) into the heat conduction equation (2), solving it for the unknown functions $v_0(r)$, $v_n(r)$, and $u_n(r)$, and then satisfying the boundary conditions, we obtain

$$v_0(r)=\tilde{A}\ln\frac{r}{a}+\int\limits_a^r\int\limits_c^{\eta}\frac{w_{02}(\xi)}{\varkappa}\,\frac{\xi}{\eta}\,d\xi d\eta,$$

$$v_n(r)=\frac{\Delta(\nu_n,a,r)}{I_0(\nu_n a)}A_n+\int\limits_a^r\frac{w_{n2}(\xi)}{\varkappa}\Delta(\nu_n,\xi,r)\xi d\xi,$$

$$u_n(r)=[\beta_n I_0(\mu_n r)+K_0(\mu_n r)+\int\limits_b^r\frac{w_{n1}(\xi)}{\varkappa}\Delta(\mu_n,\xi,r)\xi d\xi,$$

where $I_j(x)$ and $K_j(x)$ are modified Bessel functions of j-th order;

$$w_{02}(r)=\frac{1}{d}\int\limits_0^d W_2(r,z)\,dz;\quad w_{n2}(r)=\frac{2}{d}\int\limits_0^d W_2(r,z)\cos\nu_n z dz;$$

$$w_{n1}(r)=\frac{\mu_n}{N_n}\int\limits_0^d W_1(r,z)\left(\cos\mu_n z+\frac{h}{\mu_n}\sin\mu_n z\right)dz;\quad N_n=\frac{\mu_n^2 d+h(1+hd)}{2\mu_n h};$$

$$\Delta(g_n,\xi,r)=I_0(g_n\xi)K_0(g_n r)-K_0(g_n\xi)I_0(g_n r);$$

$$\beta_n=[\mu_n K_1(\mu_n b)+hK_0(\mu_n b)]/[\mu_n I_1(\mu_n b)-hI_0(\mu_n b)].$$

To determine the unknown coefficients $\tilde{A}$, A_n, and B_n of the expansion of T(r, z) in the functional series, let us use the conjugate conditions for r = c. The use of the method of re-expansion results in an infinite system of the second kind:

$$R_n+\sum_{s=1}^{\infty}G_{ns}R_s=\frac{T_0-T_1}{c\mu_n^2\ln\dfrac{c}{a}}+\tilde{W}_n,\quad n=1,2,3,\ \ldots, \tag{10}$$

where

$$R_n=N_n[\beta_n I_1(\mu_n c)-K_1(\mu_n c)]B_n;\quad G_{ns}=\frac{\gamma_s}{N_s}\sum_{m=1}^{\infty}\theta_{nms};$$

$$\gamma_s=\frac{\beta_s I_0(\mu_s c)+K_0(\mu_s c)}{\beta_s I_1(\mu_s c)-K_1(\mu_s c)};$$

$$\theta_{nms}=\frac{h}{d}\left[\frac{2\nu_m}{(\mu_n^2-\nu_m^2)(\mu_s^2-\nu_m^2)}\,\frac{\Omega(\nu_m,a,c)}{\Delta(\nu_m,a,c)}-\frac{\delta_{1m}}{c\mu_n^2\mu_s^2\ln\dfrac{c}{a}}\right];$$

$$\Omega(g_n,\xi,r)=I_0(g_n\xi)K_1(g_n r)+K_0(g_n\xi)I_1(g_n r);$$

$$\tilde{W}_n = N_n \int_b^c \frac{w_{n1}(\xi)}{\varkappa} \Omega(\mu_n, \xi, c)\xi d\xi - \frac{1}{\mu_n^2 c \ln\frac{c}{a}} \int_c^a \int_c^\eta \frac{w_{02}(\xi)}{\varkappa} \frac{\xi}{\eta} d\xi d\eta -$$

$$- \sum_{m=1}^{\infty} \frac{\nu_m}{\mu_n^2 - \gamma_m^2} \int_c^a \alpha_m(\xi) \frac{w_{n2}(\xi)}{\varkappa} \xi d\xi - \sum_{m,s=1}^{\infty} \theta_{nms} \int_b^c \frac{w_{n1}(\xi)}{\varkappa} \Delta(\mu_s, \xi, c)\xi d\xi;$$

$$\alpha_m(\xi) = \frac{\Omega(\nu_m, a, c)}{\Delta(\nu_m, a, c)} \Delta(\gamma_m, \xi, c) - \Omega(\nu_m, \xi, c);$$

and δ_{1m} is the Kronecker symbol.

The unknown coefficients A and A_n are determined as a result of simple matrix operations over B_n.

3. Let us note that the two members in the right side of (10) reflect the twin nature of the heat sources: the convective and radiative energy losses by the walls and the heating because of the microwave power.

The quantity $(\mu_n^2 - \nu_m^2)^{-1}$ in the expression for the matrix operator of the system (10) grows rapidly with the increase in n and m, since

$$\mu_{n-1} = \frac{\pi n}{d} + \frac{h}{\pi n} + O\left(\frac{1}{n^3}\right), \quad n \gg 1. \tag{11}$$

However, starting with the numbers $n \gg hd/\pi$, the process of growth of this quantity ceases. It reaches its greatest value for $n - 1 = m$, where this maximum is independent of the number n:

$$\max_{(m,\,n)} [(\mu_n^2 - \nu_m^2)^{-1}] = \frac{d}{2h}. \tag{12}$$

Using the asymptotics of the modified Bessel functions for large values of the argument, and also (11)-(12), it can be shown that

$$|G_{ns}| \leqslant \begin{cases} \dfrac{1}{s(n^2 - s^2)} \ln\dfrac{s}{n}, & n \neq s, \\ n^{-3}, & n = s. \end{cases} \tag{13}$$

The estimates (11)-(13) presented permit us to establish that

$$\lim_{M \to \infty} \sup_{(s)} \sum_{n > M} |G_{ns}| = 0,$$

i.e., the matrix operator (10) is completely continuous in the space of number sequences l_1 and, therefore ([10]), the system allows the application of the method of reduction. Hence, $\sum_n |A_n| < \infty$, $\sum_n |B_n| < \infty$; i.e., $\tilde{A}$, A_n, and B_n belong to the class of sequences satisfying the condition of finiteness of the temperature in an arbitrary region of the specimen being heated.

In the case of a discrete energy pumping mode in the cavity, the solution of the problem posed is made complicated. However, the desired temperature field can be represented as the superposition of two functions. The first is responsible for the nonstationarity of the mode, and the second is the solution of the problem examined in this paper. It is not difficult to seek the first function [9].

The magnitude of the intrinsic noise radiation intensity of the cavity walls depends on their temperature (and the energy coefficient of electromagnetic field attenuation therein) and is determined by an expression from [7]. The diffraction characteristics of the side surfaces of a biconical cavity are known. We define its wall temperature as the mean temperature with respect to the volume of all elementary inhomogeneities from which the oscillatory system is composed. The heat field of the inner washers is represented by the relations (9). The temperature distribution in the extreme washers comprising the inhomogeneity is determined analogously by using the expansion of the desired function T(r, z) in an eigenfunction series of the Sturm—Liouville problem.

Therefore, the values found for the electrodynamic and thermal characteristics of a biconical cavity permit the determination of the greatest achievable level of microwave working power, the thermal rupture modes of the system, and also the correction to the magnitude of the cavity field because of the intrinsic fluctuating thermal radiation of the heated walls. The method proposed for the computation of the electromagnetic and thermal fields of a biconical cavity by using its partition into an approximate profile of elementary inhomogeneities affords the possibility of finding the designated characteristics of a number of microwave units with arbitrary shape of the functional elements.

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UNSTEADY HEAT LOSSES OF UNDERGROUND PIPELINES

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UDC 536.24.02

Analytic expressions are presented for the unsteady temperature distribution of the ground and heat losses of an underground pipeline for an arbitrary variation of the temperature of the medium being transferred and boundary conditions of the third kind at the pipe wall and the surface of the ground.

1. The design and operation of oil and gas pipelines require calculating the heat losses of a pipeline under unsteady heat-transfer conditions. Transient thermal processes arise in oil and gas pipelines in turning off oil heating stations and devices for air cooling of gas, stopping the transfer, starting up the pipeline, etc. These processes lower the performance of the system, increase the power expended, and may lead to fusion of the rust-inhibiting insulation, a loss of longitudinal stability, and emergency stopping of transfer. To develop recommendations for ensuring reliable operation of gas and oil pipelines it is necessary to have available relations for calculating unsteady heat losses of pipelines.

Solutions of the problem of unsteady heat transfer between an underground pipeline and the surrounding medium have been obtained under a number of simplifying assumptions. A correlation of the papers on this problem is given in [1]. The most general result for large-diameter pipelines not far below the surface of the ground was obtained in [2]. However, the solution is given in the form of a double sum over eigenfunctions, which complicates its

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 2, pp. 339-345, August, 1977. Original article submitted May 27, 1976.

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